Three-Valued Logics

Notation: p, q, r atomic formulas; A, B, C arbitrary formulas
Values = {T, F, N}
Valuation = assignment of a value from Values to each atomic formula

Can be extended to an assignment of values to each formula by inductive definition using truth tables…

Truth Tables:

Bochvar = Weak Kleene = Halldén

~	
T	F
N	N
F	T

&	T	N	F
T	T	N	F
N	N	N	N
F	F	N	F

v	T	N	F
T	T	N	T
N	N	N	N
F	T	N	F

⊃	T	N	F
T	T	N	F
N	N	N	N
F	T	N	T

≡	T	N	F
T	T	N	F
N	N	N	N
F	F	N	T

Strong Kleene = Körner

~	
T	F
N	N
F	T

&	T	N	F
T	T	N	F
N	N	N	F
F	F	F	F

v	T	N	F
T	T	T	T
N	T	N	N
F	T	N	F

⊃	T	N	F
T	T	N	F
N	T	N	N
F	T	T	T

≡	T	N	F
T	T	N	F
N	N	N	N
F	F	N	T

Łukasiewicz

~	
T	F
N	N
F	T

&	T	N	F
T	T	N	F
N	N	N	F
F	F	F	F

v	T	N	F
T	T	T	T
N	T	N	N
F	T	N	F

⊃	T	N	F
T	T	N	F
N	T	**T**	N
F	T	T	T

≡	T	N	F
T	T	N	F
N	N	**T**	N
F	F	N	T

'Determinately true' operator

d	
T	T
N	F
F	F

Three ways to define validity:

1. preservation of truth (Łukasiewicz, Bochvar, Kleene, Tye):
   Γ |= B iff for every valuation v, if every formula in Γ is T on v, then B is T on v
   corresponds to designating T

2. preservation of non-falsity (Halldén):
   Γ |= B iff for every valuation v, if every formula in Γ is T or N on v, then B is T or N on v
   corresponds to designating both T and N

3. preservation of truth and non-falsity:
   Γ |= B iff for every valuation v, if every formula in Γ is T on v, then B is T on v, *and*
   if every formula in Γ is T or N on v, then B is T or N on v
   corresponds to preservation of degree (in ordering T > N > F)

Continuum-valued Logics

Values = [0,1] (set of reals from 0 to 1 inclusive)
Valuation = an assignment of values from Values to atomic formulas.

Semantics for connectives:

Łukasiewicz

$|{\sim}A| = 1 - |A|$

$|A \vee B| = \max(|A|, |B|)$

$|A \,\&\, B| = \min(|A|, |B|)$

$|A \supset B| =$ 1 if $|A| \leq |B|$
$1 – (|A|-|B|)$ if $|A| > |B|$

$|A \equiv B| = 1 – (\max(|A|,|B|) – \min(|A|,|B|))$

Goguen

$|A \,\&\, B| = |A| \times |B|$

$|A \supset B| =$ 1 if $|A| \leq |B|$
$|B| / |A|$ if $|A| > |B|$

Edgington

non-degree functional proposal

$|{\sim}A| = 1 - |A|$

$|A\&B| = |A| \times |B \text{ given } A|$

$|AvB| = |A| + |B| - |A\&B|$

Three ways to define validity:

1. preservation of perfect truth (Williamson, Edgington):
   $\Gamma \models B$ iff for every valuation v, if every formula in Γ is 1 on v, then B is 1 on v
   corresponds to designating 1

2. preservation of degree of truth (Machina):
   $\Gamma \models B$ iff for every valuation v, the value of B on v $\geq$ the lowest value assigned to a member of Γ on v
   corresponds to ordering the values and saying that a valid argument can't take you lower

3. conclusion can have no more falsity than the premises combined:
   $\Gamma \models B$ iff for every valuation v, $1-|B|_v \leq \sum_{A\in\Gamma}(1-|A|_v)$
   cf. Edgington p. 307, but note that this is not how she *defines* validity. (She uses (1).)