The Slingshot Argument

Philosophy 142—John MacFarlane

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Rules for $\Box$

We assume that the following rules are valid for the $\Box$ operator:

Substitution of logical equivalents (Equiv) If ϕ and ψ are logically equivalent, then $\ulcorner \Box\psi \urcorner$ may be inferred from $\ulcorner \Box\phi \urcorner$.

Substitution of codenoting terms (Coden) From $\ulcorner t_1 = t_2 \urcorner$ and ϕ, where t_1 and t_2 are terms (individual constants or definite descriptions), ϕ^{t_2/t_1} may be inferred, where ϕ^{t_2/t_1} is the result of substituting t_2 for some or all of the occurrences of t_1 in ϕ.

Gödel equivalence (Gödel) From $\ulcorner \Box\Phi\alpha \urcorner$, $\ulcorner \Box(\alpha = \imath x(x = \alpha \wedge \Phi x)) \urcorner$ may be inferred, and vice versa (where Φ is a predicate and α a term).

The Church slingshot

1	$P \wedge Q$	
2	$\quad \Box P$	
3	$\quad \Box(a = \imath x(x = a \wedge P))$	Equiv, 2
4	$\quad \imath x(x = a \wedge P) = \imath x(x = a \wedge Q)$	(provable from 1)
5	$\quad \Box(a = \imath x(x = a \wedge Q))$	Coden, 3, 4
6	$\quad \Box Q$	Equiv, 5
7	$\quad \Box Q$	
8	$\quad \vdots$	(as above)
9	$\quad \Box P$	
10	$\Box P \equiv \Box Q$	$\equiv$ Intro, 2–9

The Gödel slingshot

1	$a \neq b \wedge Fa \wedge Gb$	
2	$\Box Fa$	
3	$\Box(a = \imath x(x = a \wedge Fx))$	Gödel, 2
4	$\imath x(x = a \wedge x \neq b) = \imath x(x = a \wedge Fx)$	(provable from 1)
5	$\Box(a = \imath x(x = a \wedge x \neq b))$	Coden 3,4
6	$\Box a \neq b$	Gödel, 5
7	$\Box(b = \imath x(x = b \wedge a \neq x))$	Gödel, 6
8	$\imath x(x = b \wedge a \neq x) = \imath x(x = b \wedge Gb)$	(provable from 1)
9	$\Box(b = \imath x(x = b \wedge Gb))$	Coden, 7, 8
10	$\Box Gb$	Gödel, 9
11	$\Box Gb$	
12	$\vdots$	(as above)
13	$\Box Fa$	
14	$\Box Fa \equiv \Box Gb$	$\equiv$ Intro, 2–13

Problems

Exercise:

1. Prove that 'P' is logically equivalent to '$a = \iota x(x = a \wedge P)$'. Use the Russellian equivalences from the handout on Generalized Quantifiers. (Remember: to prove logical equivalence, you need to prove a biconditional using no premises.)

2. Show that (4) in the proof of the Church slingshot can be derived from (1) using standard logical rules and Russell's definition. (Note that the definite descriptions take narrow scope.)

3. In standard first-order logic it is assumed that every individual constant (e.g. 'a' in the proof above) denotes some object in the domain. That is why the inference from 'Fa' to '$\exists xFx$' is valid. Suppose we gave up this assumption. We might then change the existential generalization rule as follows:

   $$\Phi a, \exists x(x = a) \;/\; \exists x \Phi x$$

   (a) What other changes would we have to make in the system (e.g. in the other quantifier rules) to make it consistent?

   (b) Could you still prove that 'P' is logically equivalent to '$a = \iota x(x = a \wedge P)$' in such a system? Justify your answer. (You may assume that the Russellian equivalence still holds.)

4. Suppose we wanted to say (against Russell) that when there is not a unique x such that Fx, '$G(\iota xFx)$' is neither true nor false. Would 'P' still be logically equivalent to '$a = \iota x(x = a \wedge P)$'? Why or why not? (Careful: In this case the Russellian equivalence could not be used. You might try thinking of rules for ι that would still be valid. What would validity mean in such a system?)